Indian J. Phys. **54B**, 523–527 (1980)

Collective oscillations and separation of wave modes

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Received 3 November 1978, in final form 23 April 1980

Abstract. The collective oscillations in plasma are known to contain wave modes which are longitudinal and transverse in nature. The wave modes in a two component drifting plasma with finite collisions have been separated. Expressions for electric and velocity fields corresponding to electromagnetic and plasma modes have been obtained. The equilibrium plasma parameters and imposed perturbations are found to govern the magnitude of the electric and velocity fields of both the wave modes. These expressions help us estimate the strength of electric field vectors. The two wave modes are affected by the ambient plasma. The dispersion properties of electromagnetic and plasma modes have been analysed.

1. Introduction

A disturbance produced by an external source current in a compressible homogeneous plasma, can, in general, propagate in two modes, the electromagnetic mode and the plasma mode. In the hydrodynamic equation of motion (Euler equation) and in the Maxwell's field equations the two modes are coupled. A simple technique for separating the fields of the external source currents into two modes was formulated by Samaddar (1964) and Chen and Chung (1965). Verma (1966) pointed out some of the inconsistencies in the formulation of Chen and Chung (1965) and extended their work to include a lossy plasma. Both of these authors, however, considered the case of one component plasma only. Chen and Chung (1966) extended their previous formulation to include a two component lossy plasma. It is the purpose of this communication to formulate the technique of mode separation in the case of two component lossy and drifting plasma. The plasma for simplicity sake will be assumed to be homogeneous, isotropic and compressible.

2. Theoretical details

Consider the case of a two component plasma consisting of electrons and singly charged ions drifting with a velocity $\mathbf{u}_0$ along x axis. We will consider the wave modes to be propagating in the direction of the plasma drift. Maxwell's curl equations for a stationary plasma can be written as (Chen and Chung 1965, 1966)

$$\nabla \times \boldsymbol{E} = \mu_0 j\omega \boldsymbol{H} - \boldsymbol{J}_m \tag{1}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_e - j\omega\epsilon_0 \boldsymbol{E} + n_0 q(\boldsymbol{v}_i - \boldsymbol{v}_e) \tag{2}$$

where $\boldsymbol{J}_m$ and $\boldsymbol{J}_e$ represent the magnetic and electric source currents In deriving equations (1) and (2) time variation of the type $\exp(-j\omega t)$ has been assumed. The drift of the medium ($u_0 \ll c$), however, affects the second Maxwell's curl equation which for a non-magnetic drifting plasma can be written as Gavrilenko *et al* (1971)

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_e - j\omega\epsilon_0 \boldsymbol{E} + n_0 q(\boldsymbol{v}_i - \boldsymbol{v}_e) + \boldsymbol{u}_0 q(n_i - n_e) \tag{3}$$

where n_0 is the equilibrium electron or ion density and n_i, n_e, v_i and v_e are small perturbation quantities

Assuming the perturbation caused by the wave to be adiabatic and using the equation of continuity for the electron and ion components of the plasma we get

$$\nabla \cdot \boldsymbol{v}_e = jp_e\beta_1/a_e^2 m_e \tag{4}$$

$$\nabla \cdot \boldsymbol{v}_i = jp_i\beta_1/a_i^2 m_i \tag{5}$$

where

$$\beta_1 = \omega/n_0[1 - u_0 k/\omega]. \tag{6}$$

The equation of motion for the electron and ion component of the plasma can be written as

$$m_e n_0 \beta_2 \boldsymbol{v}_e - m_e n_0 \nu_e \boldsymbol{v}_i + q n_0 \boldsymbol{E} + \nabla p_e = 0 \tag{7}$$

$$-m_i n_0 \nu_i \boldsymbol{v}_e + m_i n_0 \beta_3 \boldsymbol{v}_i + \nabla p_i - q n_0 \boldsymbol{E} = 0 \tag{8}$$

where

$$\left.\begin{aligned} \beta_2 &= \nu_e - jn_0\beta_1 \\ \beta_3 &= \nu_i - jn_0\beta_1 \end{aligned}\right\} \tag{9}$$

Equations (7) and (8) are solved for $\boldsymbol{v}_i$ and $\boldsymbol{v}_e$ to get

$$\boldsymbol{v}_e = m_e\nu_e(qn_0\boldsymbol{E} - \nabla p_i) - m_i\beta_3(qn_0\boldsymbol{E} + \nabla p_e) \tag{10}$$

$$\boldsymbol{v}_i = \frac{m_e\beta_2(qn_0\boldsymbol{E} - \nabla p_i) - (qn_0\boldsymbol{E} + \nabla p_e)m_i\nu_i}{\beta_4} \tag{11}$$

where

$$\beta_4 = m_e m_i n_0(\beta_2\beta_3 - \nu_i\nu_e). \tag{12}$$

Taking curl of equation (3) and substituting for $\boldsymbol{\nabla}\times\boldsymbol{v}_e$ and $\boldsymbol{\nabla}\times\boldsymbol{v}_i$ with the help of equations (10) and (11), we get

$$\boldsymbol{\nabla}^2\boldsymbol{H}+\omega^2\mu_0\epsilon_0\epsilon\boldsymbol{H} = -\boldsymbol{\nabla}\times\boldsymbol{J}_e+\frac{\boldsymbol{\nabla}(\boldsymbol{\nabla}\,\boldsymbol{J}_m)}{\mu_0 j\omega}-j\omega\epsilon\epsilon_0\boldsymbol{J}_m \tag{13}$$

where

$$\epsilon = 1+(\omega^2 p_i+\omega^2 p_e)n_0\beta_1/\omega(\beta_2\beta_3-\nu_i\nu_e) \tag{14}$$

Equation (13) is an inhomogeneous equation whose solution can be used to determine the electric and velocity fields in the electromagnetic mode.

For obtaining the electric and velocity fields associated with the electro magnetic mode we write

$$\boldsymbol{E} = \boldsymbol{E}_m+\boldsymbol{E}_p$$

$$\boldsymbol{v}_i = \boldsymbol{v}_{mi}+\boldsymbol{v}_{pi}$$

$$\boldsymbol{v}_e = \boldsymbol{v}_{me}+\boldsymbol{v}_{pe} \tag{15}$$

where

$$\boldsymbol{\nabla}\times\boldsymbol{E}_p = \boldsymbol{\nabla}\times\boldsymbol{v}_{pi} = \boldsymbol{\nabla}\times\boldsymbol{v}_{pe} = 0.$$

Using equation (3) together with equations (10) and (11) we get, after some straight forward arithmetic

$$\boldsymbol{E}_m = j[\boldsymbol{\nabla}\times\boldsymbol{H}-\boldsymbol{J}_e]/\omega\epsilon_0\epsilon. \tag{16}$$

Having determined $\boldsymbol{E}_m$ from equation (16) we now express the velocity fields for the electromagnetic mode in terms of $\boldsymbol{E}_m$

$$\boldsymbol{v}_{me} = (m_e\nu_e q n_0-m_i\beta_3 q n_0)\boldsymbol{E}_m/\beta_4 \tag{17}$$

$$\boldsymbol{v}_{mi} = q n_0(m_e\beta_2-m_i\nu_i)\boldsymbol{E}_m/\beta_4 \tag{18}$$

In order to determine the pressure field we get from equations (3), (7) and (8)

$$\nabla^2 p_e+K_{11}p_e+K_{12}p_i = -qn_0\boldsymbol{\nabla}\cdot\boldsymbol{J}_e/j\omega\epsilon_0 \tag{19}$$

$$\nabla^2 p_i+K_{21}p_e+K_{22}p_i = qn_0\boldsymbol{\nabla}\cdot\boldsymbol{J}_e/j\omega\epsilon_0 \tag{20}$$

where

$$K_{11} = [n_0\beta_2 j\beta_1-\omega^2 p_e]/a_e^2 \tag{21}$$

$$K_{12} = [\omega^2 p_i-m_e\nu_e j n_0\beta_1/m_i]/a_i^2 \tag{22}$$

$$K_{21} = [\omega^2 p_e-m_i n_0\nu_i j\beta_1/m_e]/a_e^2 \tag{23}$$

The electric and velocity fields for the plasma mode are obtained from equations

$$K_{22} = [n_0\beta_3 j\beta_1-\omega^2 p_i]/a_i^2. \tag{24}$$

The electric and velocity fields for the plasma mode are obtained from equations (3), (10) and (11) and can be written as

$$\boldsymbol{E_p} = Q_{11}\nabla p_e + Q_{12}\nabla p_i + \boldsymbol{R}_{11}p_e + \boldsymbol{R}_{12}p_i \tag{25}$$

$$\boldsymbol{v_{pe}} = Q_{21}\nabla p_e + Q_{22}\nabla p_i + \boldsymbol{R}_{21}p_e + \boldsymbol{R}_{22}p_i \tag{26}$$

$$\boldsymbol{v_{pi}} = Q_{31}\nabla p_e + Q_{32}\nabla p_i + \boldsymbol{R}_{31}p_e + \boldsymbol{R}_{32}p_i \tag{27}$$

where

$$\left.\begin{aligned} Q_{11} &= -n_0^2 q m_i \beta_1/\omega\epsilon\epsilon_0\beta_4 \\ Q_{12} &= n_0^2 q m_e \beta_1/\omega\epsilon\epsilon_0\beta_4 \\ \boldsymbol{R}_{11} &= jq\boldsymbol{u}_0/\epsilon\epsilon_0\omega m_e a_e^2 \\ \boldsymbol{R}_{12} &= -jq\boldsymbol{u}_0/\omega\epsilon\epsilon_0 a_i^2 m_i \end{aligned}\right\} \tag{28}$$

$$\begin{aligned} Q_{31} &= j\omega m_i n_0(j\nu_i/\omega - \omega^2 p_i/\omega^2)/R' \\ Q_{32} &= n_0 m_e\omega(\omega^2 p_e/\omega^2 - j\beta_2/\omega)/jR' \\ \boldsymbol{R}_{31} &= jm_i n_0^2 q^2\boldsymbol{u}_0(m_e\beta_2/\omega m_i - \nu_i/\omega)/R' a_e^2 m_e \epsilon_0 \\ \boldsymbol{R}_{32} &= n_0^2 q^2\boldsymbol{u}_0[j\nu_i/\omega - jm_e\beta_2/\omega m_i]/R' a_i^2\epsilon_0 \end{aligned} \tag{29}$$

$$\begin{aligned} Q_{21} &= n_0 m_i\omega[\omega^2 p_i/\omega^2 - j\beta_3/\omega]/jR' \\ Q_{22} &= m_e n_0\omega[\omega^2 p_e/\omega^2 - j\nu_e/\omega]/jR' \\ \boldsymbol{R}_{21} &= \omega^2 p_e n_0 m_i\boldsymbol{u}_0[jm_e\nu_e/m_i\omega - j\beta_3/\omega]/a_e^2 R' \\ \boldsymbol{R}_{22} &= \omega^2 p_i\boldsymbol{u}_0 n_0 m_e[\nu_e/j\omega - m_i\beta_3/j\omega m_e]/a_i^2 R' \end{aligned} \tag{30}$$

and

$$R' = m_e n_0^2 m_i \epsilon[\beta_2\beta_3 - \nu_i\nu_e] \tag{31}$$

All the above equations reduce to the corresponding equations of Chen and Chung (1966) if we put $\boldsymbol{U}_0 = 0$.

3. Discussion and Conclusion

We have separated the coupled equations of hydrodynamics and those of electromagnetic field into two sets each describing just one mode From an analysis of equations (19) and (20) we find that in a plasma wave the electrons and ions are coupled This is true even if $\nabla \boldsymbol{J}_e = 0$ Thus plasma waves are made up of coupled ion and electron pressure waves. Further the equations describing the acoustic mode do not contain the magnetic source current term showing that the plasma waves in an isotropic, compressible and drifting plasma can not be generated by a magnetic source current. Further if $\nabla.\boldsymbol{J}_e = 0$ we find that plasma mode equations do not involve terms containing electric source current and therefore can be produced by electric source currents only if $\nabla\ \boldsymbol{J}_e \neq 0$.

Further if the magnetic source current is zero, we find from equation (13)

$$\Delta^2 \boldsymbol{H} + \omega^2 \mu_0 \epsilon_0 \epsilon \boldsymbol{H} = -\nabla \times \boldsymbol{J}_e \tag{32}$$

From equation (32) we find that the electromagnetic mode can still be produced if $\nabla \times \boldsymbol{J}_e \neq 0$ i e. if $\boldsymbol{J}_e \neq \Delta\alpha$, where α is a scalar.

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